Interactive long wave spectrum for the thermodynamic model

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(Manuscript received February 25, 1988; in final form June 1, 1988)

RESUMEN

El espectro analítico de absorción de Smith (1969) para la atmósfera se incorpora en un modelo termodinámico.

Esta formulación radiativa, que se aplica a la onda larga, calcula por separado la absortividad por bióxido de carbono y por vapor de agua, como función de la presión, la temperatura y el contenido de gas en la atmósfera, y lo hace con alta resolución en longitud de onda.

El agua precipitable o contenido de H_2O se calcula, usando la fórmula de Adem (1967), como función de variables evaluadas en el modelo: la temperatura superficial, la temperatura de la troposfera media y la extensión horizontal de la nubosidad.

Con este enfoque el modelo puede simular el efecto de retroalimentación positiva debido a la opacidad infrarroja del vapor de agua; es decir, su efecto invernadero.

El espectro calculado para los valores actuales de las concentraciones de CO_2 y H_2O concuerda bien con las estimaciones de Goody y Robinson (1951), Goody (1954) y Fleagle y Businger (1963).

ABSTRACT

Smith's (1969) analytical absorption spectrum of the atmosphere is incorporated in a thermodynamic model.

This radiative formulation is applied to the infrared region. It computes separately the absorptivity by carbon dioxide and by water vapor, with a high wave length resolution, as a function of atmospheric pressure, temperature and gas content.

The precipitable water or H_2O content is computed using Adem's (1967) formula as a function of variables evaluated in the model: the surface temperature, the mid-tropospheric temperature and the horizontal extent of cloudiness.

With this approach the model is able to simulate the positive feedback effect by long wave opacity of water vapor; that is to say its greenhouse effect.

The computed spectrum for present values of CO_2 and H_2O concentrations shows good agreement with the estimates of Goody and Robinson (1951), Goody (1954) and Fleagle and Businger (1963).

1. Introduction

In a recent version of Adem's thermodynamic model (Adem and Garduño, 1982, 1984) we used an atmospheric absorption spectrum, constructed by assuming that, for long wave radiation, the atmosphere:

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i.   behaves as a black body (that is, wholly opaque) in the wave length intervals $(0,8\mu m)$ and $(18\mu m, \infty)$,

ii.   has a window in the interval $(8, 12 \ \mu m)$ and,

iii.   between 12 and $18\mu m$, behaves according to the CO_2 content; we assume that in this interval the only atmospheric component which absorbs is the CO_2; its $15\mu m$ band is approximated by six steps of $1\mu m$ wide, whose height (absorptivity) is a function of the CO_2 content and is determined using the results of Yamamoto and Sasamori (1958, 1961) who computed curves of absorptivity against frequency, and gas content, for different values of pressure and temperature.

This parameterization of the infrared spectrum allowed us to vary the CO_2 content in the modeled atmosphere and simulate its greenhouse effect. In previous papers we applied this approach to evaluate the climate alteration induced by doubling the atmospheric CO_2 concentration (Adem and Garduño, 1982, 1984).

In such numerical experiments we did not take into account the effect on the absorptivity of changes in atmospheric pressure and temperature during the CO_2 doubling process. This effect seems to be small (Yamamoto and Sasamori, 1958, 1961). Furthermore, we had assumed that H_2O does not emit radiation in the interval $(12, 18 \ \mu m)$, and has a fixed spectrum during the whole experiment.

The purpose of this paper is to introduce Smith's (1969) formula to parameterize the long-wave spectrum, in order to introduce a variable emission by H_2O from 12.5 μm on, besides taking the CO_2 radiatively active in the interval $(12.2, 19.8 \ \mu m)$. In this approach, the spectrum is computed analytically by means of an empirical formula deduced by Smith (1969), which gives absorptivity as a function of gas content, and equivalent temperature and pressure, defined as vertical averages weighted with the gas distribution in the atmosphere. Furthermore Adem (1967) derived a method, not incorporated in his model up till now, to evaluate the amount of precipitable water in the atmosphere using climate variables computed in the model: the surface temperature, the mid-tropospheric temperature and the horizontal amount of cloudiness.

Using together both parameterizations, Smith (1969) for the spectra and Adem (1967) for precipitable water, we are able to generate in the model the interaction between atmospheric water vapor and climate, via long wave radiation.

Other authors have dealt with the water vapor problem in a different way. Manabe and Wetherald (1967) assume that the climate system keeps constant the relative humidity of the atmosphere. They also fix the cloudiness. In a later paper Manabe and Wetherald (1975), assuming fixed cloudiness, incorporate a hydrological cycle and an interactive lapse rate. As for long wave radiation, they use gray spectra, interactive with climate, and applied to a multiple-layer atmosphere.

Ramanathan (1976) uses an infrared band spectrum, i.e., with some resolution in wave length. His model has also many atmospheric layers. The model of Cerni and Parish (1984) uses a gray spectrum and many layers. Liou (1980) gives formulas for both options: gray and band spectra.

Paltridge (1974), Sellers (1973, 1976), Petukhov (1974) and Sergin and Sergin (1976) incorporate in their models partial hydrological cycles.

For an exhaustive list of models and their features, refer to the very complete and up to date Schlesinger's (1984) review.

2. Analytical spectrum

Smith (1969) fitted experimental based transmission data of the $15\mu m$ CO_2 and rotational H_2O bands with a polynomial formula. The spectral variable is the wave number (n) and the coefficients are determined with 5 cm^{-1} resolution. The validity intervals of the formula are (505, 820 cm^{-1}) for CO_2 and (200, 800 cm^{-1}) for H_2O. The formula for the H_2O absorptivity is

$$a = 1 - exp\{-exp(C_o + C_1 ln\frac{T_o U}{T U_o} + C_2 ln\frac{P}{P_o} + C_3 ln\frac{T}{T_o} +$$

$$C_4 ln\frac{T_o U}{T U_o} ln\frac{P}{P_o} + C_5 ln\frac{T_o U}{T U_o} ln\frac{T}{T_o} + C_6 ln^2\frac{T_o U}{T U_o} + C_7 ln^2\frac{T_o U}{T U_o} ln\frac{T}{T_o} + C_8 ln\frac{P}{P_o} ln^2\frac{T}{T_o})\} \qquad (1)$$

where a is the fractional absorptivity, P the pressure, T the (absolute) temperature, $T_o = 273K$, $P_o = 1013$ mb and $U_o = 1$ cm, U is the gas content (also called optical mass or path length), which is measured in cm as the thickness of the gas when it is liquid at STP conditions, namely at temperature T_o and pressure P_o. For H_2O, U is the precipitable water. The formula for CO_2 is similar to the one for H_2O, and it is obtained from (1) by substituting the terms with the coefficients C_7 and C_8 by the following expression

$$C_7 ln^2\frac{T_o U}{T U_o} ln\frac{P}{P_o} + C_8 ln\frac{T_o U}{T U_o} ln^2\frac{T}{T_o}. \qquad (2)$$

The coefficients C_i $(i = 0,...,8)$ depend on the gas and the wave number, and are given by Smith (1969) for H_2O and CO_2.

About the application of formulas (1) and (2) to an environment inhomogeneous in T or P (as the atmosphere), these variables should be taken with their equivalent values, defined as the weighted means with respect to the gas content through the layer thickness:

$$\overline{T} = \frac{\int_o^U T dU}{\int_o^U dU} \qquad (3)$$

$$\overline{P} = \frac{\int_o^U P dU}{\int_o^U dU} \qquad (4)$$

where the superimposed bar indicates the equivalent value, dU is the differential of gas content, and

$$U = \int_o^U dU. \qquad (5)$$

In the next sections we will evaluate the integrals in (3), (4) and (5) in the tropospheric layer. We will use the same vertical distributions of temperature and pressure, as Adem (1962), which are given by the formulas:

$$T = T_a - \beta z \qquad (6)$$

$$P = P_a(1 - \frac{\beta z}{T_a})^\gamma \qquad (7)$$

where T and P are the values of temperature and pressure at height z (upward coordinate with origin at the Earth's surface, taken as sea level), T_a and P_a are, respectively, the values of T and P at $z = 0$; $\gamma = g/R\beta$ where β is the lapse rate, g the acceleration of gravity and R the gas constant of the air. We don't take into account orography, therefore, the bottom of the troposphere is taken at the sea level. This level is indicated with subindex a.

In these computations we will deal only with typical values, i.e., global and annual averages of the variables: $T_a = 288K$, $P_a = P_o$, $h = 9\,km$, $\beta = 6.5K\,km^{-1}$, $g = 980cm\,s^{-2}$, $R = 2.87 \times 10^6 cm^2 s^{-2} K^{-1}$.

When both gases absorb in the same wave number, the net absorptivity is computed assuming that their transmissivities should be multiplied (Smith, 1969). Then the combined absorptivity is

$$a_{AB} = 1 - (1 - a_A)(1 - a_B) \tag{8}$$

where A refers to H_2O and B to CO_2.

The parameterization of the long wave radiation used previously (Adem, 1962, 1982; Adem and Garduño, 1982) has the wave length (λ) as spectral variable, while Smith (1969) uses the wave number (n). Both variables are related by

$$n\lambda = 1. \tag{9}$$

Therefore, it is necessary to transform Smith's spectrum from the variable n (in cm^{-1}) to λ (in μm). In order to accomplish this, we divide the n coordinate equidistantly in λ; commonly by intervals of 1 μm, as is used in the model. Of course, according to (9), each interval has a different number (not integer, in general) of 5 cm^{-1} (Smith's resolution) steps.

Initially we compute the absorptivity as a function of n by means of formula (1) and expression (2), from 5 to 5 cm^{-1}; then we get the value of a in each $1\mu m$ wide interval in λ as simple average of the corresponding 5 cm^{-1} step values that enter in the $1\mu m$ interval; the value of a step that enters incomplete in the interval is taken proportional to the fraction included.

3. CO_2 Parameters

According to Elsasser (1940) the amount of water vapor above approximately the 300 mb level is negligible small. Therefore, it is realistic to assume that the effective radiation temperature of the upper boundary of the model's atmospheric layer is at about this level. For this reason we use as the hight of the layer a value $h = 9\,km$, and the corresponding pressure $P_h = 307.3\,mb$. Therefore, we will compute the atmospheric long wave spectrum taking into account only the CO_2 contained in the modeled layer of hight h. This amount will be denoted U_B.

Smith (1969) assumes that the vertical distribution of CO_2 is homogeneous in pressure, namely

$$\frac{dU_B}{dP} = c \tag{10}$$

where c is a constant. Then

$$c = \frac{U_B}{P_a - P_h} \tag{11}$$

where P_a and P_h are the values of the pressure at the surface and at the height h, respectively.

In order to compute the equivalent pressure $(\overline{P}_B)$ for the CO_2 contained in the model's tropospheric layer, we use (10) in formula (4) and include only this layer. The result is

$$\overline{P}_B = \frac{P_a + P_h}{2}. \tag{12}$$

For the total atmospheric CO_2 content (U_{B_T}) we assume the value 260 cm, which is very close to the value used previously (Adem and Garduño, 1982, 1984).

As U_B is contained between P_a and P_h, and U_{B_T} between P_a and 0, according to (10): $U_B = 181.1$ cm. Therefore from (11) and (12), $c = 0.2567$ cm mb^{-1} and $\overline{P}_B = 660.15$ mb, respectively.

Using (6), (7) and (10) in formula (3), and evaluating the integrals in the model's tropospheric layer, we get the equivalent temperature of the CO_2

$$\overline{T}_B = \frac{cT_h}{U_B(\alpha + 1)}\left(\frac{P_a^{\alpha+1}}{P_h^\alpha} - P_h\right) \tag{13}$$

where $\alpha = R\beta/g = 1/\gamma$.

Therefore, $\overline{T}_B = 263.43$ K.

4. Precipitable water

The amount of precipitable water or H_2O content in the layer of hight h is given by

$$U_A = \int_{T_h}^{T_a} u_{A_T}(T)dT \tag{14}$$

where $u_{A_T}(T)$ is the vertical distribution of H_2O content as a function of the temperature.

Adem (1967) derived a formula to evaluate $u_{A_T}(T)$, which is the following:

$$u_{A_T}(T) = \frac{R_o}{\rho_a}(A + B\epsilon + \alpha + \alpha_1 T + \alpha_2 T^2)(\frac{a'}{T} + b' + c'T + d'T^2 + g_1 T^3) \tag{15}$$

where

$$R_o = \frac{0.622}{R\beta}$$

$$\alpha = A_1\frac{T_a}{\beta} + A_2(\frac{T_a}{\beta})^2$$

$$\alpha_1 = -\frac{1}{\beta}(A_1 + 2A_2\frac{T_a}{\beta})$$

$$\alpha_2 = \frac{A_2}{\beta^2}.$$

The constant a', b', c', d', g_1, A, A_1, A_2 and B are empirical parameters (Adem, 1967), which come from climatology and laboratory; some of them depend on the season, but here we deal only with typical values; i.e., annual averages. Their values are:

$a' = 1.252 \times 10^7 \text{gr s}^{-2}\text{cm}^{-1}$

$b' = -1.997 \times 10^5 \text{gr s}^{-2}\text{cm}^{-1}\text{K}^{-1}$

$c' = 1.196 \text{gr s}^{-2}\text{cm}^{-1}\text{K}^{-2}$

$d' = -3.19 \text{gr s}^{-2}\text{cm}^{-1}\text{K}^{-3}$

$g_1 = 3.2 \times 10^{-3} \text{gr s}^{-2}\text{cm}^{-1}\text{K}^{-4}$

$A = 0.4518$

$A_1 = -8.23 \times 10^{-7}\text{cm}^{-1}$

$A_2 = 5.5 \times 10^{-13}\text{cm}^{-2}$

$B = 0.5$.

Formula (14) with $u_{A_T}(T)$ given by (15), shows that U_A is a function of T_a, T_h, β and ϵ; besides, according to (6), $\beta = (T_a - T_h)/h$.

Using observed normal values of T_a, T_h, ϵ and h, Adem (1967) computed the Northern Hemisphere distributions of U_A for winter and summer and compared them successfully with the observed corresponding values.

Formula (14), gives U_A as the mass of water contained in an atmospheric column with a base of unit area; then, in cgsK system its units are gr cm^{-2}. An alternative method is to measure U_A as the volume, which the water contained in the same column would occupy if it were liquid at STP conditions, divided by the unit area; in this case units of U_A are cm. Since STP density of liquid water is $\rho_a = 1$gr cm^{-3} both methods to measure precipitable water are numerically the same in cgsK system. We prefer U_A in cm.

5. H_2O parameters

The vertical distribution function of H_2O content can be expressed as a function of z, T or P. This distribution is defined as

$$u_{A_r}(r) = \frac{dU_A}{dr} \tag{16}$$

where r is z, T, or P, $u_{A_r}(r)$ is the vertical distribution of H_2O expressed as a function of r, and dU_A is the element of H_2O content between coordinate levels r and $r + dr$, corresponding to U_A and $U_A + dU_A$, respectively.

From (16) in order to change the variable of $u_{A_T}(T)$ from T to z, it is necessary to multiply the second member by $\frac{dT}{dz}$. Therefore,

$$u_{A_z}(z) = u_{A_T}(T) \frac{dT}{dz}. \tag{17}$$

Using (6) we obtain:

$$u_{A_z}(z) = -\beta u_{A_T}(T). \tag{18}$$

This is negative because U_A decreases with z.

Now we will compute the equivalent temperature $(\overline{T}_A)$ and the equivalent pressure $(\overline{P}_A)$ for H_2O in the model's tropospheric layer.

From equation (3) and (16), with $r = T$ we obtain

$$\overline{T}_A = \frac{1}{U_A} \int_{T_h}^{T_a} T u_{A_T}(T) dT \tag{19}$$

where U_A is given by (14).

From (6) and (7) we get

$$P = P_h (\frac{T}{T_h})^\gamma. \tag{20}$$

Substituting (20) in Eq. (4) and using (16) with $r = T$ we obtain

$$\overline{P}_A = \frac{P_a}{U_A T_a^\gamma} \int_{T_h}^{T_a} T^\gamma u_{A_T}(T) dT. \tag{21}$$

6. Numerical experiments

a) Normal vertical distribution of atmospheric H_2O

As basic normal values for present conditions we take $\epsilon = 0.5$ and T_a, β and h as used in section 2.

Using these values in formulas (15) and (18) we obtain the normal vertical distribution of the H_2O content $(-u_{A_z})$ as a function of height, which is shown by the continuous line of Fig. 1.

To compare the computed values of u_{A_z} with observed ones we use the formula

$$u_{A_z} = \frac{\rho q}{\rho_a} \tag{22}$$

where ρ is the density of humid air and q the specific humidity. As usual, we assume ρ the same as for dry air, and

$$q = m \tag{23}$$

where m is the mass mixing ratio. Similarly, we take R the same for dry and moist air. The data used are from Houghton (1979), and Peixóto and Oort (1984). Houghton gives m as a function of latitude and pressure coordinate; we take the values corresponding to 30° N in latitude, inasmuch as this parallel is representative of the Hemisphere, by dividing it in two equal areas. Peixóto and Oort give q also as a function of latitude and pressure; we take again the values for 30° N, because this latitude is the mean position of the 2.5 cm isoline of precipitable water which is the global average. Observed u_{A_z} is evaluated with (22), taking ρ from Holton (1979).

In Fig. 1 the encircle dots and crosses correspond to the Houghton-Holton and Peixóto and Oort-Holton observed values of u_{A_z}, respectively, which agree quite well with the computed values.
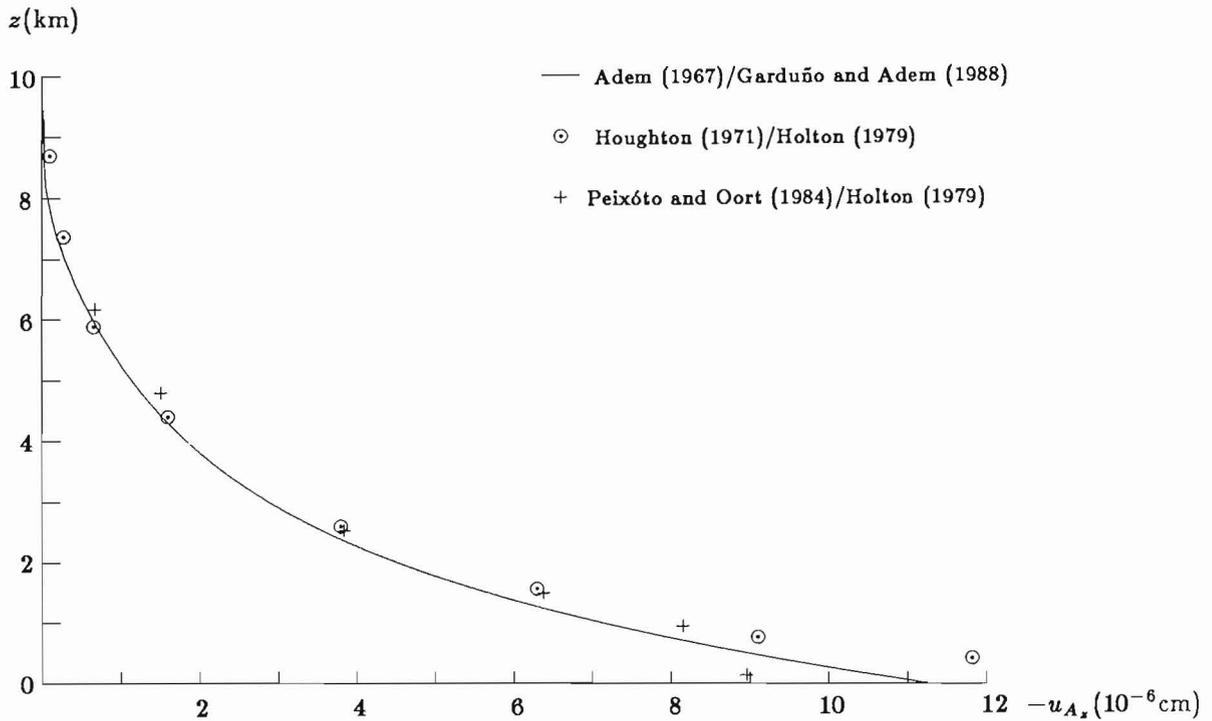
Fig. 1. The H_2O content in the atmosphere $(-u_{A_z})$ as a function of height (z). The continuous line corresponds to the computed values using formulas (15) and (18). The encircle dots and crosses correspond to the Houghton (1979) - Holton (1979) and Peixóto and Oort (1984)- Holton (1979) observed values, respectively.

b) Normal value of precipitable water

Using the computed $u_{A_T}(T)$ in formula (14) we obtain

$$U_A = 2.4 \text{ cm}$$

this result is in good agreement with the observed value of U_A which is equal to 2.5 cm (Peixóto and Oort, 1984).

c) Normal atmospheric spectra

We define a spectrum as the graph of absortivity against wave length. Here we will show the present climate spectra for H_2O, for CO_2 and for their combination as it was explained in Sec. 2.

The computations are carried out in the following way:

Using formula (15) in (19) and (21), we obtain $\overline{T}_A = 275.55$ K and $\overline{P}_A = 816.28$ mb, respectively.

In order to evaluate the absorptivities (a_A and a_B) due to H_2O and CO_2, we apply formula (1) and expression (2) with the corresponding parameters. We use the computed values of U_A, $\overline{T}_A$ and $\overline{P}_A$, and those of U_B, $\overline{T}_B$ and $\overline{P}_B$, for U, T and P, respectively. The coefficients C_i are given by Smith (1969), who lists them as functions of the wave number, n, for each 5 cm^{-1} interval. $a_A(n)$ and $a_B(n)$ are transformed to become functions of the wave length (λ) in the way explained at the end of Sec. 2.
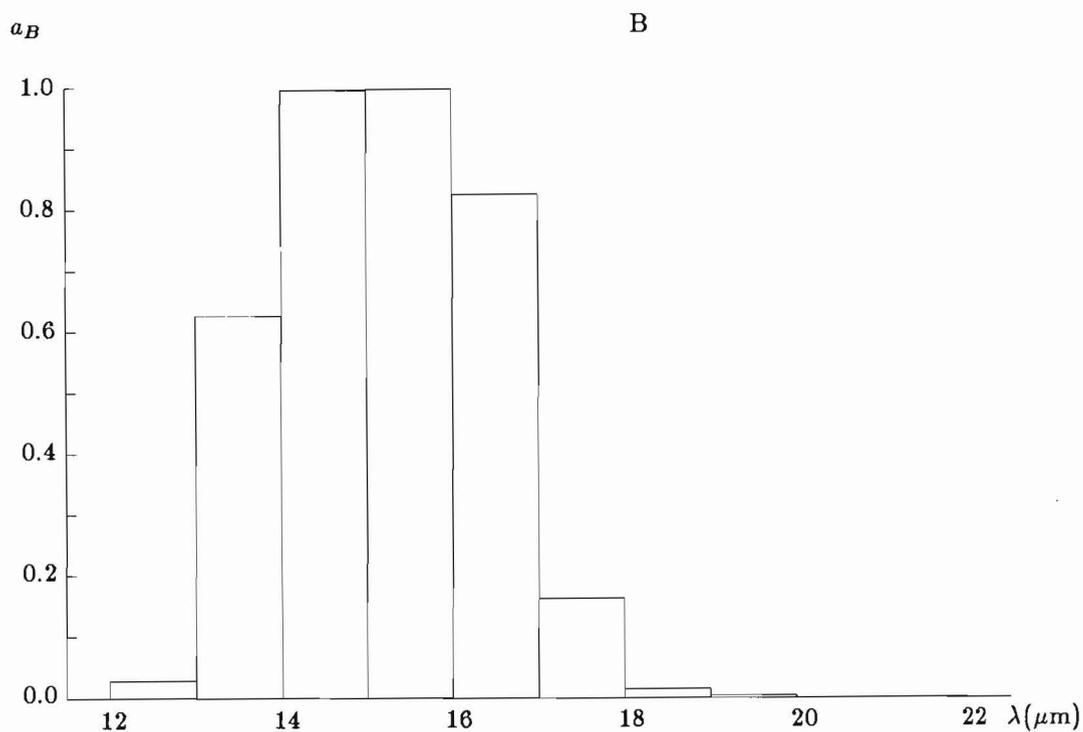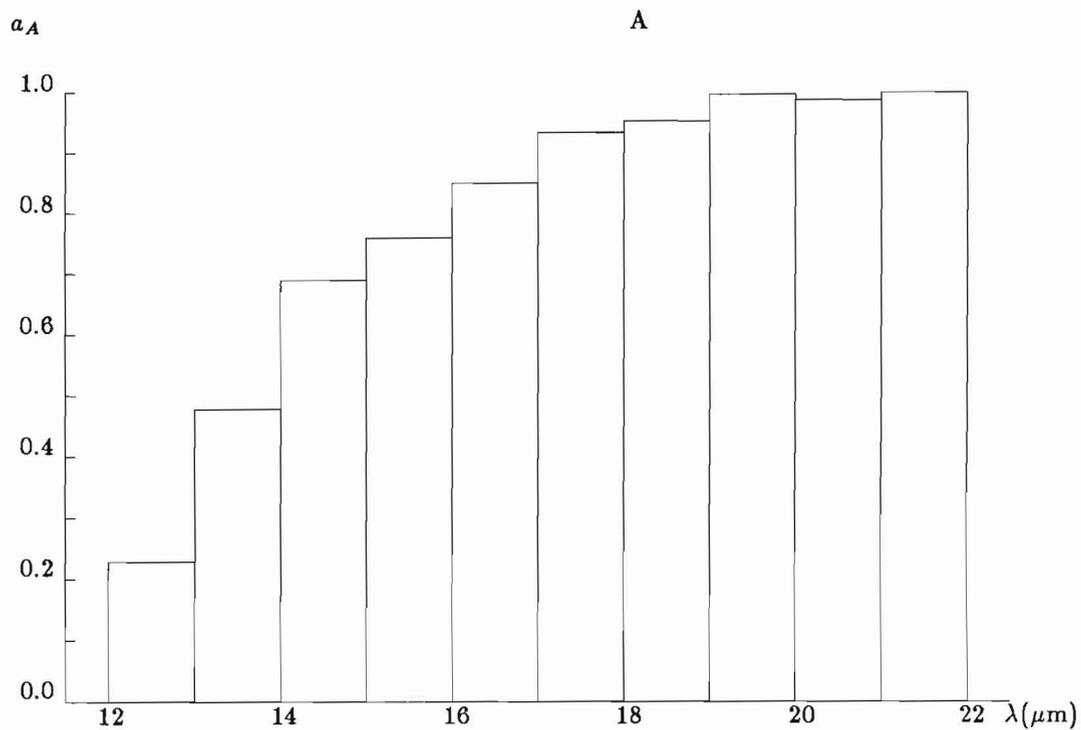
Fig. 2. Computed normal atmospheric absorptivity as a function of wave length, with $1\mu m$ resolution. For H_2O (part A), CO_2 (part B) and $H_2O + CO_2$ (part C).
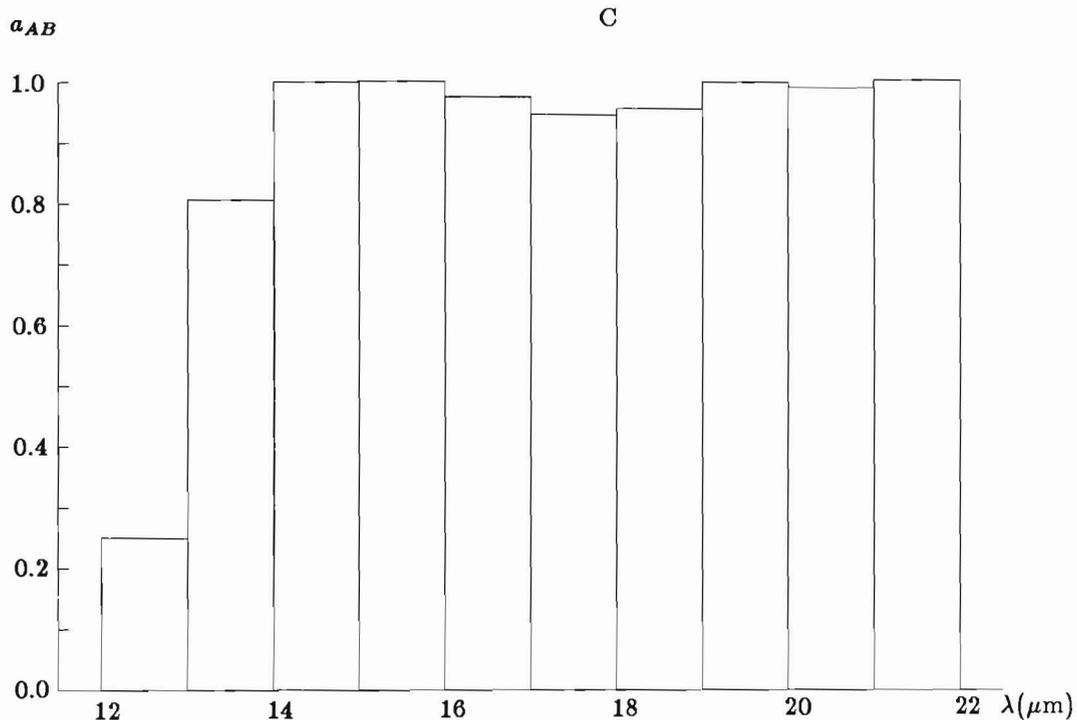
Fig. 2 (continued)

$\lambda(\mu m)$	a_A	a_B	a_{AB}
12.5	0.2285	0.0272	0.2495
13.5	0.4781	0.6265	0.8050
14.5	0.6899	0.9954	0.9986
15.5	0.7592	0.9983	0.9996
16.5	0.8493	0.8249	0.9736
17.5	0.9325	0.1625	0.9435
18.5	0.9515	0.0139	0.9521
19.5	0.9953	0.0028	0.9953
20.5	0.9864	0.0000	0.9864
21.5	0.9990	0.0000	0.9990

TABLE I. a_A, a_B and a_{AB} are the present climate absoptivities due to H_2O, CO_2 and $H_2O + CO_2$, respectively, corresponding to the wave length $\lambda(\mu m)$ at intervals of one micrometer.

The validity intervals of Smith's (1969) formulas are $(12.5, 50\mu m)$ for H_2O and $(12.2, 19.8\mu m)$ for CO_2. We will compute the absorptivity from $\lambda = 12\mu m$ on. In order to complete a one-micrometer step in the cases when there is a gap in the interval, we assume that the absorptivity is equal to zero in such gap; e.g., for the step $(12, 13\mu m)$, we use $a_B = 0$ in $(12, 12.2\mu m)$.

Once evaluated $a_A(\lambda)$ and $a_B(\lambda)$ for each one-micrometer step, we compute the combined absorptivity $a_{AB}(\lambda)$ for the gas mixture by means of formula (8).

a_A and a_B are computed in the intervals $(12, 50\mu m)$ and $(12, 20\mu m)$, respectively. For $\lambda > 20\mu m$ we assume $a_B = 0$. Furthermore, from (1) we obtain $a_A = 1$ for $\lambda > 22\mu m$. Therefore the combined spectrum is also saturated in the same region. Formula (1) and expression (2) are not applicable for $\lambda < 12\mu m$. For this reason, we assume as in previous papers (Adem, 1962; Adem and Garduño, 1982) that for $8 < \lambda < 12\mu m$, $a_{AB} = 0$; and that for $\lambda < 8\mu m$, $a_{AB} = 1$. Therefore the relevant spectral results discussed in this paper are in the interval $(12, 22\mu m)$.

The results of the computations are shown in Table I where the computed absorptivities due to H_2O, CO_2 and $H_2O + CO_2$ are shown in the second, third and fourth columns respectively. The values correspond to the wave length at intervals of one micrometer identified by their central values, shown in the first column.

Figure 2 shows the values of Table I. Parts A, B and C are the absorptivities due to H_2O, CO_2 and $H_2O + CO_2$, respectively. In the three parts of the figure the abscissa is the wave length in micrometers and the ordinate is the absorptivity.

d) Comparation with other spectra

We looked for normal atmospheric spectra reported by other authors in order to compare them with that of Fig. 2C.

After a wide search we found only three different spectra:

A: The one computed by Goody and Robinson (1951).

B: The one, by Goody (1954).

C: The one given by Fleagle and Businger (1963).
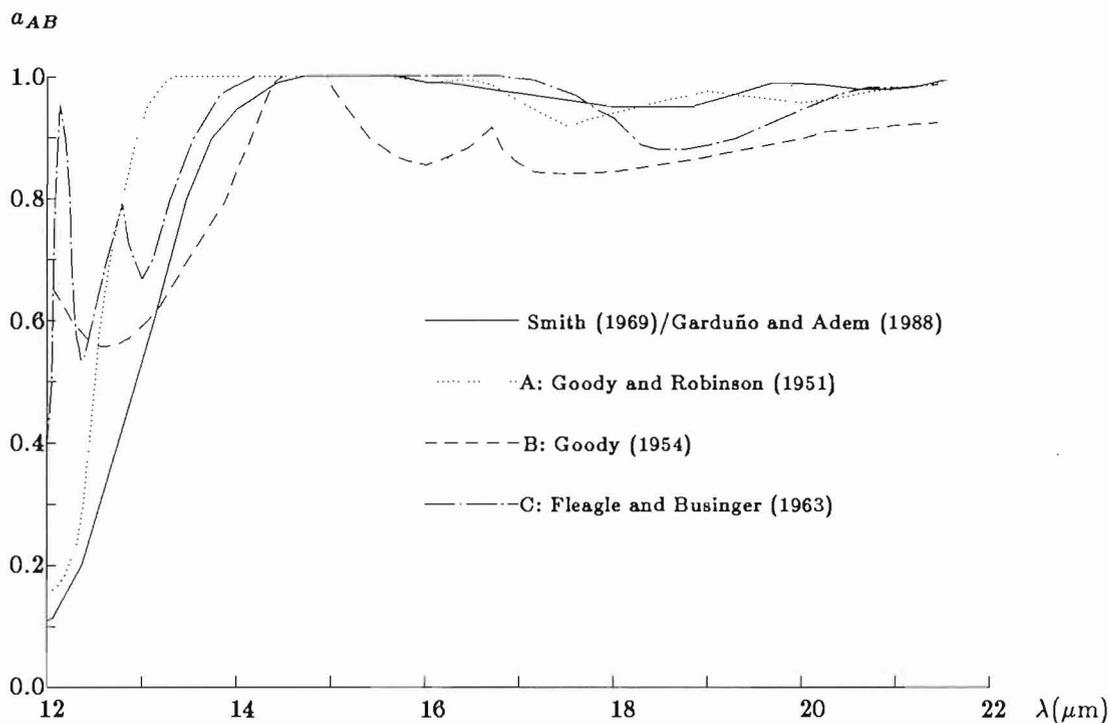
Fig. 3. Atmospheric spectra due to $H_2O + CO_2$. Computed by our method (continuous line) by Goody and Robinson, 1951 (dotted line), Goody, 1954 (dashed line), and Fleagle and Businger, 1963 (dashed-dotted line).

The spectrum B is reproduced by Goody and Walker (1972). Fleagle and Businger (1963) report the spectrum C without a clear information about its origin. They mention the work of Goody and Robinson (1951), but spectra A and C are not the same. Kondratyev (1969) reproduces the spectrum A, attributing it to Goody (1954) instead of Goody and Robinson (1951). Houghton (1977) has an illustration of spectrum B without explaining its source.

The spectra A, B and C are shown in Fig. 3, together with the spectrum of Fig. 2C, which is plotted as a curve instead of a histogram, in the interval from 12 to $21\mu m$. This figure shows that anyone of the four spectra does not coincide with any other anywhere, except between 14.5 and $15\mu m$, where all of them are saturated. We can see also that our spectrum is in very good agreement with the spectrum A for $\lambda \geq 15\mu m$, and in good agreement with the spectrum C between 13 and $14.5\mu m$, and again with spectrum A between 12 and $12.5\mu m$.

Spectrum A is lower than ours from $15\mu m$ on, probably because Goody and Robinson (1951) computed it with an amount of 2 cm of water in the atmosphere, instead of the value 2.4 cm used by us. For $\lambda \geq 15\mu m$ the main absorber is H_2O as can be seen in Fig. 2, and by reducing the H_2O content the spectrum is lowered as will be illustrated bellow. Goody and Robinson (1951) do not give data about the amount of CO_2 used in calculating the spectrum A.

The information given by Goody and Walker (1972), and by Fleagle and Businger (1963) about the spectra B and C, respectively, is not enough to explain discrepancies.

e) Spectra sensitivity

Now we illustrate the sensitivity of our spectra with respect to the gases (H_2O and CO_2) content in the atmosphere. In order to show also the improvement obtained with an increment in the wave length resolution, we compute these spectra with steps of $0.5\mu m$. These are plotted in Figs. 4 and 5
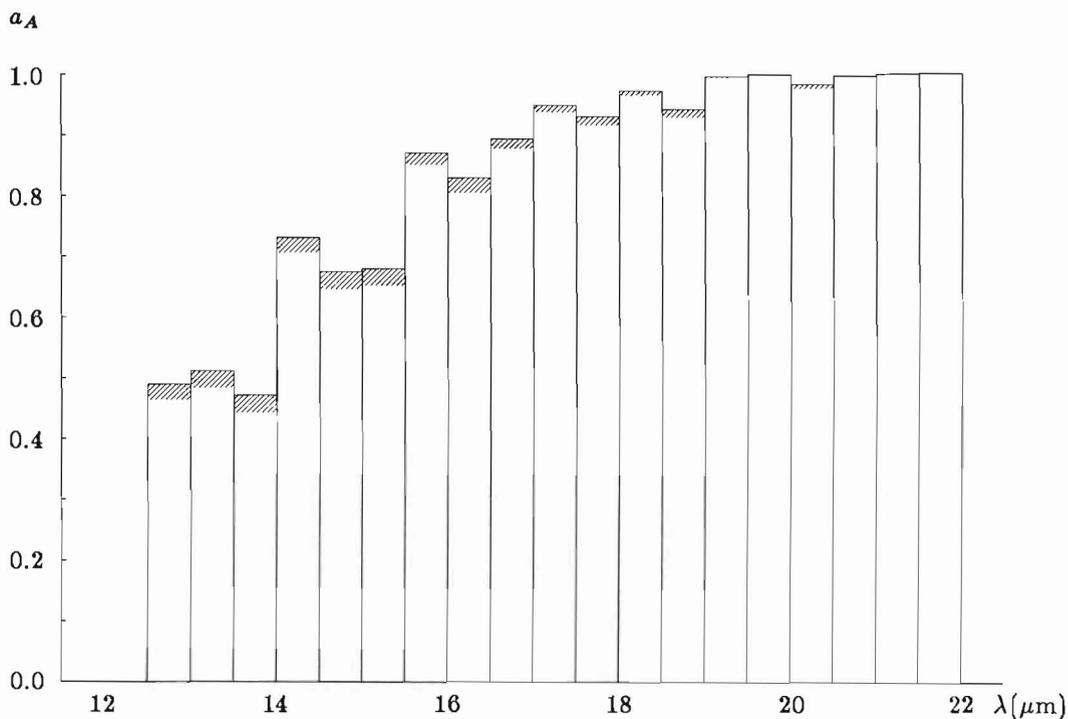
Fig. 4. Computed spectra for H_2O, with $0.5\mu m$ resolution. The upper histogram corresponds to 2.50 cm, and the lower one to 2.25 cm of H_2O. The hatched area indicates the difference between both cases.

for H_2O (i.e., the graph of a_A against λ) and for CO_2 (i.e., a_B vs λ), respectively.

The spectra of Fig. 4 correspond to two cases: the absorptivity due to 2.5 cm of precipitable water (upper histogram) and for 2.25 cm (lower one). The value of $U_A = 2.5$ cm is the actual average (Peixóto and Oort, 1984), thus we illustrate the effect of diminishing the H_2O content by 10%. The hatched area is the respective spectrum decrement. Both cases are computed with $\overline{P}_A = 816.28$ mb and $\overline{T}_A = 275.55$ K, which are the values determined in subsection 6c.

Similarly, Fig. 5 shows the spectrum of CO_2 for its actual content (lower histogram) and for the duplicated one (upper histogram). The values of these contents were taken as $U_B = 261$ and 522 cm, which are the same ones used by us in previous papers (Adem and Garduño, 1982, 1984). In both cases we used the values $\overline{P}_B = 660.15$ mb and $\overline{T}_B = 263.43$ K, which were determined in Sec. 3. The spectral increment due to the doubling of CO_2, which is the hatched area, is equal to 0.374μm.
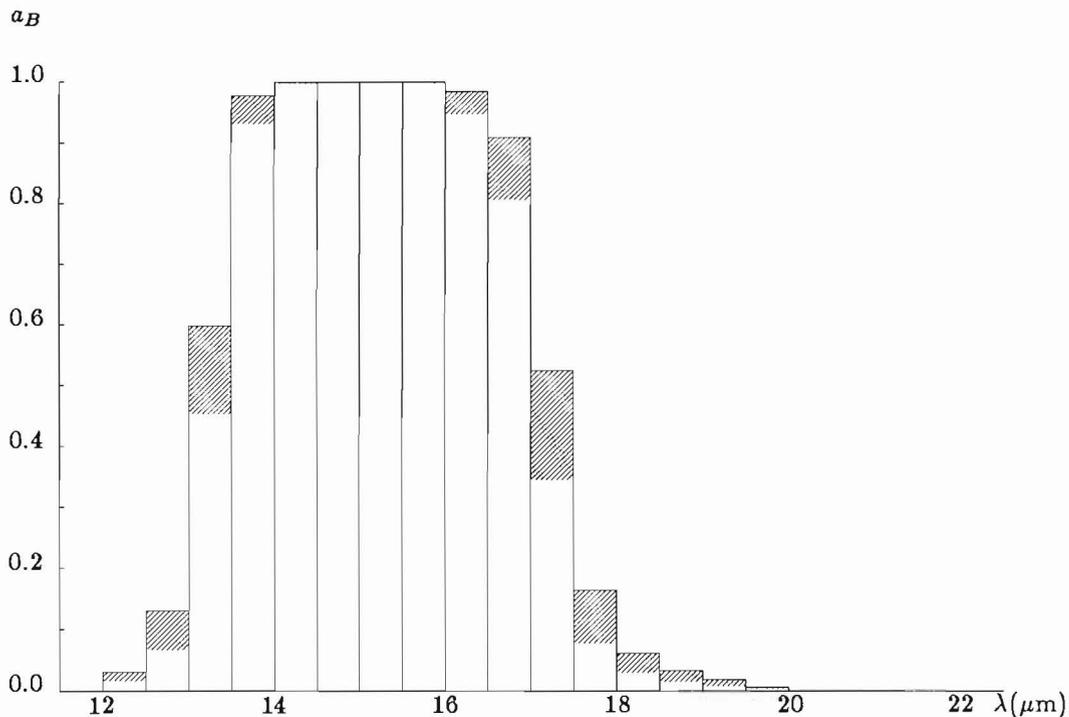
Fig. 5. Computed spectra for CO_2, with 0.5μm resolution, using Smith's (1969) approach. The lower histogram corresponds to 261 cm of CO_2, and the upper one to 522 cm. The hatched area, which indicates the difference between both cases, is equal to 0.374μm.

Besides the spectra of Fig. 5 (with 0.5μm resolution), we show in Fig. 6 the corresponding spectra (with 1μm resolution) which were computed previously by us (Adem and Garduño, 1982), using the results of Yamamoto and Sasamori (1958, 1961). The spectral increment due to the increase of the CO_2 content from 261 to 522 cm, which is the hatched area, is in this case equal to 0.303μm.

Comparison of the CO_2 spectra of Figs. 5 and 6 shows the possible improvement due to Smith's (1969) formulation, which allows to use a finer resolution (doubled in this illustration, but capable of more detail yet). The greater resolution is able to detect better the sensitivity of the absorption band wings, which are the lowest and consequently the most alterable zones of the spectrum. Probably this is the main reason why the increment in absorptivity due to the CO_2 duplication is larger in our new spectrum (from Smith, 1969) than in the previous one (from Yamamoto and Sasamori, 1958, 1961). The difference is equal to 0.071μm, which represents a 23% increase.
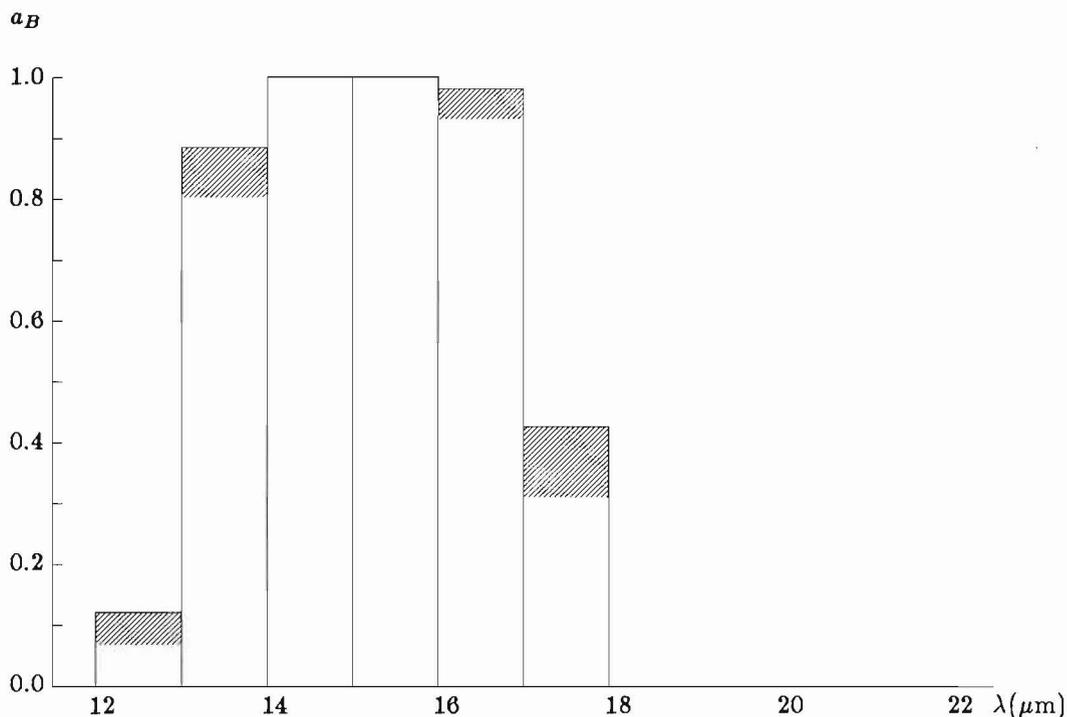
Fig. 6. Computed spectra for CO_2, with 1.0μm resolution, using Yamamoto and Sasamori's (1958, 1961) approach. The hatched area, which indicates the difference between both cases, is equal to 0.303μm.

Although its comparison with another one is not illustrated, the spectrum of H_2O has similar advantages than that of CO_2.

7. Final remarks and conclusions

An attempt has been made to develop a parameterization of an interactive long wave spectrum due to the atmospheric contents of water vapor and carbon dioxide adequate for use in a thermodynamic climate model.

The numerical computations show that the spectrum for present climate is realistic and that it is also sensitive to changes in the H_2O and CO_2 contents. Furthermore, the parameterization allows to express the spectrum in terms of the surface temperature, the horizontal extent of cloudiness, and the mid-tropospheric temperature, variables which are included in the climate thermodynamic model. Therefore we hope that such parameterization can be used in such a model to simulate in a more realistic way the climate. Preliminary experiments (Garduño and Adem, 1987, 1988) have shown that this is possible. However more complete experiments, using this parameterization, to compute the effect of an increase of the atmospheric CO_2 content and a change in the solar constant, will be carried out. The parameterization will also be used to possibly improve the simulation of the climates from the last glaciation, 18,000 years ago, to present (Adem, 1988).

This spectral formulation should be extended for $\lambda < 12\mu$m, which is a region of great thermal importance for climate. Similarly, it is necessary to have absorptivity formulas for other atmospheric components, mainly O_3, although it would be essential in this case, to extend the climate model to include at least a stratospheric layer.

Acknowledgements

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