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*SHORT CONTRIBUTION*

# The dependence of the power–law exponent on surface roughness and stability in a neutrally and stably stratified surface boundary layer

N. M. ZOUMAKIS

*Laboratory of Atmospheric Physics, Technological Education Institute (T. E. I.), P. O. Box 14561, Thessaloniki 54101, Greece*

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RESUMEN

La dependencia del exponente ($p$) de la ley exponencial del perfil del viento de la rugosidad superficial y de la estabilidad atmosférica se describe suponiendo una variación con la altura de $\phi$ (Huang, 1979; Hanafusa *et al.*, 1986) y empleando la formulación de Zilitinkevich (1989) para establecer los perfiles de velocidad en una capa limítrofe planetaria neutra y establemente estratificada.

Las estimaciones teóricas del exponente de la ley exponencial son perfectamente comparables con los datos de dicha ley provenientes de varias fuentes y de los análisis teóricos de Irwin (1979). Se reconoce que se necesita más trabajo antes de que la validez de la metodología propuesta pueda ser plenamente establecida.

ABSTRACT

The dependence of the wind profile power-law exponent ($p$) on surface roughness and atmospheric stability is depicted assuming a height variation of $p$ (Huang, 1979; Hanafusa *et al.*, 1986) and using the formulation of Zilitinkevich (1989) for determining the velocity profiles in a neutrally and stably stratified planetary boundary layer. The theoretical estimates of the power–law exponent compare perfectly with the power–law exponent data from various sources and the theoretical analysis from Irwin (1979). It is recognized that more work is necessary before the validity of the suggested methodology can be fully established.

## 1. Introduction

The magnitude and direction of near–surface winds and their variations with height, in the planetary boundary layer, are of considerable interest in both research and applied studies, such as Air Pollution (Stern, 1976; Hanna *et al.*, 1982), Building Design (Sachs, 1978; Melaragno, 1982) and Wind Energy Technology (Justus *et al.*, 1976; Gourieres, 1982). Since most of the available wind speed measurements have been made close to the ground, it is necessary to extrapolate the wind speed profiles within the surface boundary layer (Segal and Pielke, 1988). The most common extrapolation is based on the power–law velocity equation, which is preferred by engineers, for mathematical simplicity, and also provided a reasonable fit to the oberved wind velocity profiles, in the lower part of the planetary boundary layer. It may be expressed as,

$$u(z)/u_1 = (z/z_1)^p, \qquad p > 0 \tag{1}$$

where $u(z)$ and $u_1$ are the wind speed at the heights $z$ and $z_1$, respectively, where the height

$z_1$ is at the reference level. The exponent $p$ is then to be determined on the basis of surface configuration and atmospheric conditions.

As discussed by Arya (1988) the logarithmic profile law has a sounder theoretical and physical basis than the power–law profile, especially within the neutral surface layer. There is no exact correspondence between the power–law and logarithmic wind profile, because the two profile shapes are different. However, the use of the power law gives a reasonably accurate representation of the wind speed profiles and a better representation than does the logarithmic law for the entire depth of the planetary boundary layer (Counihan, 1975; Hanafusa *et al.*, 1986; Segal and Pielke, 1988). On the other hand, the power–law exponent, $p$, is found to depend on the surface roughness, atmospheric stability and height range over which the power–law is fitted to the observed profile. Following Panofsky and Prasad (1965) and Panofsky and Dutton (1983), Equation (1) can be rearranged to give,

$$p = (z/u)(\partial u/\partial z) \tag{2}$$

Given the dimensionless wind shear which is a function of $\varsigma = z/L$ only,

$$\partial u/\partial z = (u_*/kz)\phi(z/L) \tag{3}$$

integration of Equation (3) gives the surface layer wind profile for the stable atmospheric conditions (Businger *et al.*, 1971; Zoumakis and Kelessis, 1991a),

$$u(z) = (u_*/k)\{\ln(z/z_o) + \beta[(z - z_o)/L]\} \tag{4}$$

where $u_*$ is the friction velocity, $k = 0.40$ is the von Karman constant, $L$ is the Monin-Obukhov stability length, $z_o$ is the surface roughness length (which is a surface characteristic of a fixed site and is not supposed to change with stability), and $\beta = 4.7$. Substitution of Equations (3) and (4) into (2) finally yields an approximate relationship:

$$p = [1 + 4.7(z/L)]/[\ln(z/z_o) + 4.7(z/L)] \tag{5}$$

The value of $p$ is computed at a height $z$; however it provides, as suggested by Huang (1979) and Sedefian (1980), a good representation for a sublayer between $z_1$ and $z_2$ when $z = (z_1 z_2)^{0.5}$. Unfortunately, the theoretical predictions of the power–law exponent from Equation (5), under stable conditions, overestimate the $p$–values (Irwin, 1979; Zoumakis and Kelessis, 1991b; Zoumakis, 1992).

## 2. Methodology and Discussion

For a comparison of the power–law and surface layer wind profiles, it was assumed that the near–surface velocity profile should satisfy the logarithmic + linear law (4); however, the region where it holds true is limited in addition to the inequality $z << h$, also by another inequality $L << h$ (Zilitinkevich, 1989), where $h$ is the stably stratified planetary boundary layer ($PBL$) depth (Delage, 1974; Wyngaard, 1975; Arya, 1977; Brost and Wyngaard, 1978; Yamada, 1979; Zilitinkevich, 1989). On the other hand, in the evening and at night with the increase of hydrostatic stability the atmospheric $PBL$ collapses; its upper boundary falls more and more, while turbulence in the overlying region degenerates (Zilitinkevich, 1989). A further analysis involved substituting the surface layer wind profile from Equation (4) by a velocity profile for the whole

$PBL$ region, $z \leq h$,

$$u(z) = (u_*/k)[\ln(z/z_o) + f_u(z/h,\ \mu)],\ v(z) = (-u_*/k)f_v(z/h,\ \mu)sign(f) \tag{6}$$

$$L = -u_*^3/k\mathbf{B}_s, \qquad \mu = ku_*/\mid f \mid L \tag{7}$$

$$h = u_*\Lambda(\mu)/\mid f \mid \tag{8}$$

where $\mu$ is the dimensionless stratification parameter, $B_s$ is the near-surface value of the vertical turbulent buoyancy flux, $f$ is the Coriolis parameter and the functional forms for $f_u(z/h,\ \mu)$, $f_v(z/h,\ \mu)$ and $\Lambda(\mu)$ are taken from Zilitinkevich (1989). On the other hand, the velocity profiles from Equations (6), (7) and (8) satisfy the logarithmic + linear law (4) and $v(z) = 0$, in the region at $z << h$ and $L << h$. Combining Equations (1), (6), (7) and (8), the wind profile power-law exponent $p$, for the whole $PBL$ region $z \leq h$, has the following form:

$$p(z_1,\ z_2,\ z_o,\ L,\ h) = \ln(U_2/U_1)/\ln(z_2/z_1) \tag{9}$$

where $U_2$ and $U_1$ are the wind vectors at the heights $z_2$ and $z_1$, respectively (Ku *et al.*, 1987). Assuming an independent height scale $h$ (Clarke and Hess, 1974; Zilitinkevich and Deardorff, 1974), Figure 1 compares the theoretical predictions for $p$, from Equation (9), with the power–

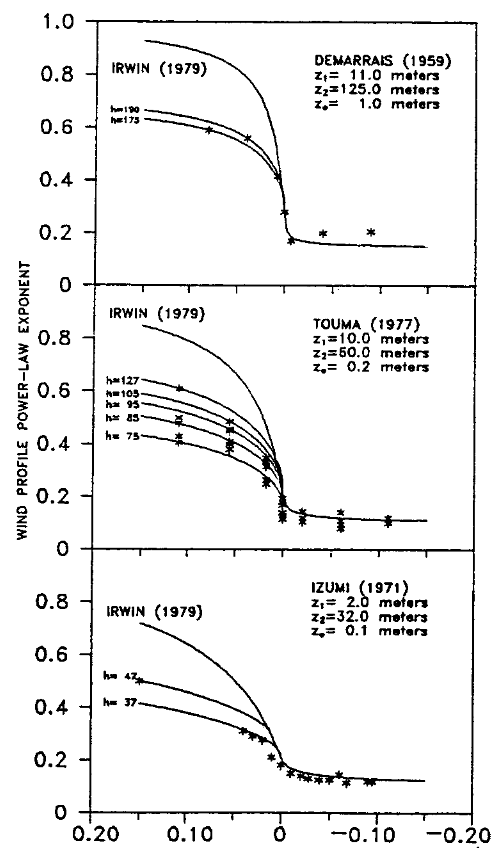


Fig. 1. Comparisons of the theoretical estimates of the power-law exponent values, from Equation (9), with experimental data from three sources and theoretical predictions from Irwin's formula.

law exponent data (De Marrais, 1959; Izumi, 1971; Touma, 1977; Irwin, 1979) and estimates from Equation (5). In Figure 1 the plots indicate that, during stable conditions, the theoretical estimates from Equation (9) compare perfectly with the data. Despite the extremely low $h$–values used in Figure 1 and the large uncertainty in determining the height scale $h$, the suggested Equation (9) seems to explain the dependence of $p$ on surface roughness and atmospheric stability, during stable conditions.

Although the mechanism is not completely clear at this stage, in view of Equation (9) and the power–law exponent data published by Irwin (1979), it appears possible that the classical Equation (5) overestimates the $p$–values. However, more theoretical studies and more data on the power–law exponent as a function of $L$ and $h$, at different Pasquill stability classes, over representative land types, will be required before the suggested procedure can be used in an operational mode. Hopefully, future work will help us to increase our understanding of the dependence of the power–law exponent on surface roughness and atmospheric stability, during stable conditions.

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### REFERENCES

Arya, S. P. S., 1988. Introduction to Micrometeorology, Academic Press, Inc., San Diego, California.

Arya, S. P. S., 1977. Suggested Versions to Certain Boundary–Layer Parameterization Schemes Used in Atmospheric Circulation Models, *Mon. Wea. Rev.*, **105**, 215-227.

Brost, R. A. and J. C. Wyngaard, 1978. A Model Study of the Stably Stratified Planetary Boundary Layer, *J. Atmos. Sci.*, **35**, 1424-1440.

Businger, J. A., J. C. Wyngaard, Y. Izumi and E. F. Bradley, 1971. Flux–Profile Relationships in the Atmospheric Surface Layer. *J. Atmos. Sci.*, **28**, 181-189.

Clarke, R. H. and G. D. Hess, 1974. Geostrophic Departure and the Functions A and B of Rossby–number Similarity Theory. *Boundary–Layer Meteorol.*, **7**, 267-287.

Counihan, J., 1975. Adiabatic Atmospheric Boundary Layer: A Review and Analysis of Data from the Period 1880-1972. *Atmos. Environ.*, **9**, 871-905.

Delage, Y., 1974. A Numerical Study of the Nocturnal Atmospheric Boundary Layer. *Quart. J. Roy. Meteor. Soc.*, **100**, 351-364.

DeMarrais, G. A., 1959. Wind Speed Profiles at Brookhaven National Laboratory. *J. Meteorol.*, **16**, 181-190.

Gourieres, D., 1982. Wind Power Plants Theory and Design, Pergamon Press Ltd. Oxford, England.

Hanafusa, T., C. B. Lee and A. K. Lo, 1986. Dependence of the Exponent in Power Law Wind Profiles on Stability and Height Interval. *Atmos. Environ.*, **20**, 2059-2066.

Hanna, S. R., G. A. Briggs and R. P. Hosker, 1982. Handbook on Atmospheric Diffusion, DOE/TIC-11223, U. S. A. Department of Energy, Washington.

Huang, C. H., 1979. A Theory of Dispersion in Turbulent Shear Flow. *Atmos. Environ.*, **13**, 453-463.

Irwin, J. S., 1979. A Theoretical Variation of the Wind Profile Power–Law Exponent as a Function of Surface Roughness and Stability. *Atmos. Environ.*, **13**, 191-194.

Izumi, Y., 1971. Kansas 1968 Field Program Data Report, AFCRL–72–0041, Hanscom AFB Massachusetts.

Justus, C. G., W. R. Hargraves and A. Yalcin, 1976. Nationwide Assessment of Potential Output from Wind Powered Generators. *J. Appl. Meteorol.*, **15**, 673-678.

Ku, J. Y., S. T. Rao and K. S. Rao, 1987. Numerical Simulation of Air Pollution in Urban Areas: Model Development. *Atmos. Environ.*, **21**, 201-212.

Melaragno, M., 1982. Wind in Architectural and Environmental Design, Van Nostrand Reinhold Company, New York.

Panofsky, H. A. and B. Prasad, 1965. Similarity theories and diffusion. *Int. J. Air Water Pollut.*, **9**, 419-430.

Panofsky, H. A. and J. A. Dutton, 1983. Atmospheric Turbulence, Wiley and Sons.

Sachs, D., 1978. Wind Forces in Engineering, Pergamon Press Ltd. Oxford, England.

Sedefian, L., 1980. On the Vertical Extrapolation of Mean Wind Power Density. *J. Appl. Meteorol.*, **19**, 488-493.

Segal, M. and R. A. Pielke, 1988. The Extrapolation of Vertical Profiles of Wind Speed within the Marine Atmospheric Surface Layer Using the $p$ Formula. *J. Appl. Meteorol.*, **27**, 174-181.

Stern, A. C., 1976. Air Pollution, Volume 1, Air Pollutants, Their Transformation and Transport, Academic Press, New York.

Touma, J. S., 1977. Dependence of the Wind Profile Power Law on Stability for Various Locations, *J. Air Pollut. Control Assoc.*, **27**, 863-866.

Wyngaard, J. C., 1975. Modeling the Planetary Boundary Layer–Extension to the Stable Case. *Boundary–Layer Meteorol.*, **9**, 441-460.

Yamada, T., 1979. CPBL Similarity Profiles Determined from a Level–2 Turbulence–Closure Model. *Boundary–Layer Meteorol.*, **17**, 333-351.

Zilitinkevich, S. S., 1989. Velocity Profiles, The Resistance Law and the Dissipation Rate of Mean Flow Kinetic Energy in a Neutrally and Stably Stratified Planetary Boundary Layer. *Boundary–Layer Meteorol.*, **46**, 367-387.

Zilitinkevich, S. S. and J. W Deardorff, 1974. Similarity Theory for the Planetary Boundary Layer of Time–Dependent Height. *J. Atmos. Sci.*, **31**, 1449-1452.

Zoumakis, N. M. and A. G. Kelessis, 1991a. The Dependence of the Bulk Richardson Number on Stability in the Surface Layer. *Boundary–Layer Meteorol.*, **57**, 407-414.

Zoumakis, N. M. and A. G. Kelessis, 1991b. Methodology of Bulk Approximation of the Wind Profile Power–Law Exponent Under Stable Stratification. *Boundary Layer Meteorol.*, **55**, 199-203.

Zoumakis, N. M., 1992. On the Relationship between the Gradient and the Bulk Richardson Number for the Atmospheric Surface Layer. *Nuovo Cimento*, **15**C, 111-114.